Test Metrics

I have tested most of these and they work, 17 Aug 2022. Does not work on accented coordinates like . Also must be changed to (e.g. see (64))

# Imaginary metrics designed to test code

Asymmetric metric

|  |  |
| --- | --- |
|  | (1) |

Does not get italicized

Caused crash in SetLinear

Caused havoc

Extreme equations

|  |  |
| --- | --- |
|  | (2) |

terms not at end

|  |  |
| --- | --- |
|  | (3) |

Too many terms in one group

|  |
| --- |
|  |

Equation ends with

|  |  |
| --- | --- |
|  | (4) |

Equation contains

|  |  |
| --- | --- |
|  | (5) |

Equation contains other

|  |  |
| --- | --- |
|  | (6) |

Equation contains does not contain enough

|  |  |
| --- | --- |
|  | (7) |

Equation contains wrong

|  |  |
| --- | --- |
|  | (8) |

Equation contains wrong

|  |
| --- |
|  |

Diagonal metric with zero diagonal component

Wrong size metric

# Euclidean metrics

**3D Euclidean**

|  |  |
| --- | --- |
|  | (9) |
|  | (10) |

**Plane polar coordinates**

|  |  |
| --- | --- |
|  | (11) |

Coordinates radial, angle to axis

|  |  |
| --- | --- |
|  | (12) |

**Spherical polar**

Coordinates radial, polar, azimuthal (= longitude)

|  |  |
| --- | --- |
|  | (13) |

|  |  |
| --- | --- |
|  | (14) |

**2-sphere: Surface of sphere (S2)**

Coordinates polar, azimuthal

|  |  |
| --- | --- |
|  | (15) |

This metric is often written

|  |  |
| --- | --- |
|  | (16) |

So is the angle subtended between two points.

**Ellipsoid, Elliptic paraboloid, Hyperbolic paraboloid**

Ellipsoid coordinates

|  |  |
| --- | --- |
|  | (17) |

|  |  |
| --- | --- |
|  | (18) |

Elliptic paraboloid coordinates

|  |  |
| --- | --- |
|  | (19) |
|  | (20) |

Hyperbolic paraboloid coordinates

|  |  |
| --- | --- |
|  | (21) |

|  |  |
| --- | --- |
|  | (22) |

**General 2D case**

with coordinates . Obviously there is no particular metric so we just say it is

|  |  |
| --- | --- |
|  | (23) |

The Riemann tensor has only one independent component. It is convenient to use for that. In all up to eight of the possible sixteen are non zero. The Ricci tensor is

|  |  |
| --- | --- |
|  | (24) |

or

|  |  |
| --- | --- |
|  | (25) |

and the scalar curvature is

|  |  |
| --- | --- |
|  | (26) |

**Poincare half plane**

The metric of the Poincare half plane is

|  |  |
| --- | --- |
|  | (27) |
|  | (28) |

**Two torus T²**

There are two kinds of tori flat and curved. They are both generated by two circles but to find the former you need to start in four dimensions. A metric for a curved torus is

|  |  |
| --- | --- |
|  | (29) |

and for a flat torus

|  |  |
| --- | --- |
|  | (30) |

In both cases are the radii of the generating circles. See [Commentary 8.2 Tori.pdf](https://drive.google.com/open?id=1Jq50s3ojRV7CBjqyb2QzP87JYLv4tJIG). The flat torus is notable because it is flat but has a finite area.

**3-sphere**

Coordinates , metric

|  |  |
| --- | --- |
|  | (31) |

**Relativistic metrics**

**Minkowski**

Coordinates

|  |  |  |
| --- | --- | --- |
|  |  | (32) |

|  |  |
| --- | --- |
|  | (33) |

**Spherical Minkowski**

Coordinates as in spherical

|  |  |
| --- | --- |
|  | (34) |

We could also write

|  |  |
| --- | --- |
|  | (35) |

where

|  |  |
| --- | --- |
|  | (36) |

We use this frequently below.

|  |  |
| --- | --- |
|  | (37) |

**Schwarzschild metric (spherical polar)**

See Carroll's 5.1. Coordinates are time, radial, polar, azimuthal

|  |  |
| --- | --- |
|  | (38) |

We often replace (twice Newton's gravitational constant times the central mass) by the Schwarzschild radius which is the radius of event horizon and the last part by . That would be

|  |  |
| --- | --- |
|  | (39) |

As matrices we have

|  |  |  |
| --- | --- | --- |
|  |  | (40) |
|  |  | (41) |

**Eddington-Finkelstein metric**

The Eddington-Finkelstein metric is for the same spacetime as Schwarzschild but with coordinates

|  |  |
| --- | --- |
|  | (42) |

where

|  |  |
| --- | --- |
|  | (43) |

if we also add

|  |  |
| --- | --- |
|  | (44) |

we can get a metric equation with coordinates

|  |  |
| --- | --- |
|  | (45) |

with implicitly defined by

|  |  |
| --- | --- |
|  | (46) |

**Kruskal predecessor**

On the way to the Kruskal metric we get coordinates with metric equation

|  |  |
| --- | --- |
|  | (47) |

|  |  |
| --- | --- |
|  | (48) |

In this coordinate system and are both always null.

**Kruskal**

Kruskal coordinates are where

|  |  |
| --- | --- |
|  | (49) |
|  | (50) |

and they give a metric equation

|  |  |
| --- | --- |
|  | (51) |

Or

|  |  |
| --- | --- |
|  | (52) |

with implicitly defined from

|  |  |
| --- | --- |
|  | (53) |

It is useful to have eliminated from the solution because it is greater than infinity inside the event horizon!

These coordinates have great advantages because they cover the whole of spacetime very regularly. is timelike everywhere and is spacelike everywhere and there are other similarities with flat Minkowski spacetime.

**de Sitter x 4**

de Sitter metrics are maximally symmetric and therefore have constant curvature. The first de Sitter metric has positive curvature:

|  |  |
| --- | --- |
|  | (54) |

or

|  |  |
| --- | --- |
|  | (55) |

de Sitter conformal with as metric on two sphere

|  |  |
| --- | --- |
|  | (56) |

or

Anti de Sitter spacetime has negative curvature and its metric is

|  |  |
| --- | --- |
|  | (57) |

or

|  |  |
| --- | --- |
|  | (58) |

Anti de Sitter conformal

|  |  |
| --- | --- |
|  | (59) |

or

|  |  |
| --- | --- |
|  | (60) |

**Robertson-Walker (FLRW)**

Other names Friedmann, Lemaitre. Carroll's equations 8.38, 8.43

|  |  |
| --- | --- |
|  | (61) |

or

|  |  |
| --- | --- |
|  | (62) |

and

|  |  |
| --- | --- |
|  | (63) |

or

|  |  |
| --- | --- |
|  | (64) |

The second version is Carroll's preferred form - 'flouting' conventional wisdom.